

EXERCISE 1: Consider the finite Markov chain $Y = \{Y_n, n \in \mathbb{N}\}$ reported in Figura 1. Complete the following task:

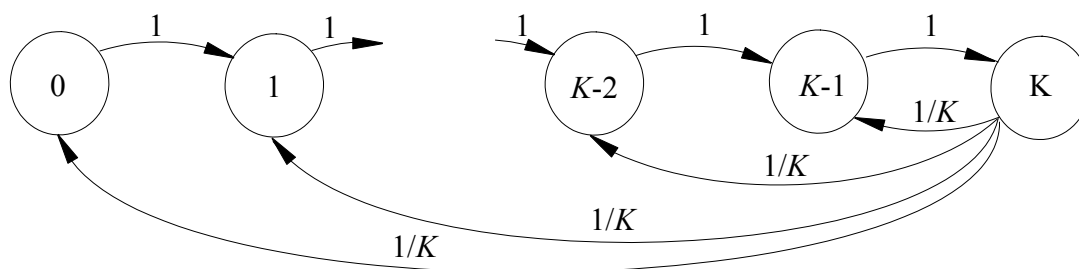


Figura 1: Diagramma delle probabilità di transizione

1. Describe the properties of the Markov chain and those of its states and calculate the stationary state probabilities $\{\pi_i, i = 0, 1, 2, \dots, K\}$, provided they exist.

Assume now that the above Markov chain $Y = \{Y_n, n \in \mathbb{N}\}$ models the service process of an M/G/1 queueing system. Specifically, assume that $\pi_k = P\{X = k\}$, $0 \leq k \leq K$ where X is a discrete random variable representing the service time. On the assumption that packets arrive with a constant rate λ , complete the following task:

2. determine the stability condition of the M/G/1 queueing system and calculate the average number of packets in the system.

Finally, assume that $Y = \{Y_n, n \in \mathbb{N}\}$ be a Markov chain modulating a process which generates packets. In other words, when the Markov chain visits state $\{k\}$, $0 \leq k \leq K$, the process emits $k + 1$ packets. Complete the following task:

3. Calculate the average number of packets $E[A]$ emitted by the process in the time unit.

