EXERCISE 1: Consider an Internet router where packets arrive according to a Poisson process of rates

$$\lambda_n = \begin{cases} \dfrac{N-n}{N(n+1)} & 0 \le n < N \\ 0 & n \ge N \end{cases}$$

which thus depend upon the number of packets $n$ present in the system. Assume that the router under examination has an output telecommunication line and indicate with $\mu$ the rate of the exponential distribution which describes the packet transmission time on that line.

Complete the following tasks:

1. draw the transition rate diagram of the M/M/1 queuing system which models the router;
2. determine the stability condition(s) and calculate, for a stable system, the stationary state probabilities;
3. calculate the distribution observed by an arriving packet;
4. calculate the throughput, the carried load and the offered load by exploiting the associated definitions;
5. calculate the average number $E[N]$ of packets in the system and the average response time $E[R]$ .

NOTE: You might find it useful to take into consideration that:

(i) $$k\binom{N}{k} = N\binom{N-1}{k-1}$$

(ii) $$\frac{N(N-1)(N-2)\cdot \ldots \cdot (N-k+1)}{k!} = \binom{N}{k}$$

(iii) $$\sum_{k=0}^{N}\binom{N}{k}\frac{1}{x^k} = \left(1+\frac{1}{x}\right)^N$$

(iv) $$\left(\frac{N-j}{j+1}\right)\binom{N}{j} = \binom{N}{j+1}$$

(v) $$k\binom{N}{k} = N\binom{N-1}{k-1}$$