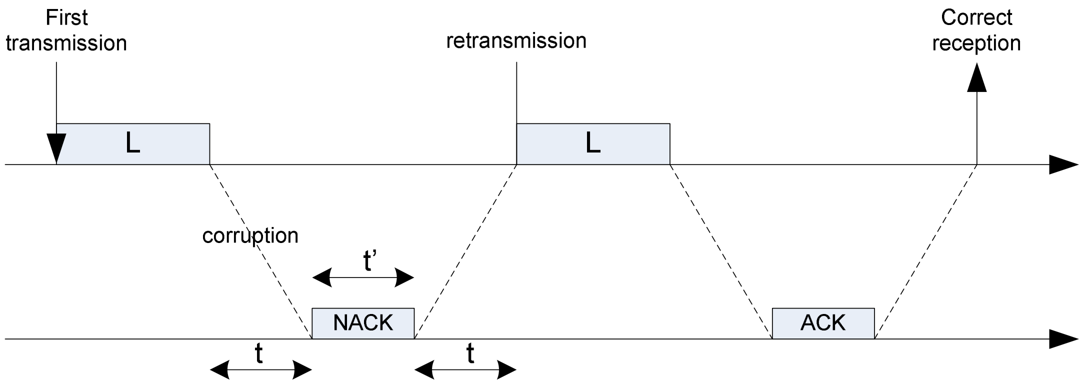
Exercise 2

Consider a MAC-layer protocol where a transmitter sends frames whose length is L bits over a line whose transmission speed is C bits per second. The receiver acknowledges the frames by sending back an ACK, if the frame has been received correctly, or a NACK, if the frame is corrupted. A frame is corrupted if at least one bit is corrupted, and the link's *bit error rate* (i.e., the probability that a *single bit* is corrupted) is constant and equal to p. The transmitter retransmits the same frame until it receives an ACK.

Assume that:

- all transmitted bits are independent of each other
- an ACK/NACK never gets corrupted
- the propagation time along both directions of the link is constant and equal to t
- the time it takes to transmit an ACK/NACK is equal to t'

1) Compute the probability p_{err} that a frame is corrupted
2) Compute the values that the RV T, "time to correct reception of a frame" can assume
3) Compute the PMF of the above RV
4) Compute the mean value of T
5) Assuming that a frame has been corrupted k-1 times, find the conditional probability that it will be corrupted at the subsequent transmission attempt. Justify the result.



Solution

1) $p_{err} = 1 - \binom{L}{0} p^0 \cdot (1-p)^{L-0} = 1-(1-p)^L$

2) T can be equal to $t_k = k \cdot (2t + t' + L/C)$, $k \geq 1$ being the number of required transmissions before an ACK is received back.

3) Call $P_j = P\{T = t_j\}$. It is $P_j = \left(\prod_{i=1}^{j-1} p_{err,i} \right) \cdot (1 - p_{err,j}) = p_{err}{}^{j-1} \cdot (1 - p_{err})$.

4) It can be easily seen that T is a (scaled) geometric variable, hence its mean value is $E[T] = (2t + t' + L/C) / (1 - p_{err})$.

5) The conditional probability is

$$P\{\text{corrupted kth time} \mid \text{corrupted k-1 times}\} = \frac{P\{\text{corrupted k consecutive times}\}}{P\{\text{corrupted k-1 times}\}}$$

$$= \frac{p_{err}{}^{k}}{p_{err}{}^{k-1}} = p_{err} = P\{\text{corrupted}\}$$

This is obvious, since retransmissions are independent of each other.