

EXCERCISE 1: Consider the Markov chain characterized by the following probability transition matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \dots \\ a & b & c & 0 & 0 & \dots \\ 0 & a & b & c & 0 & \dots \\ 0 & 0 & a & b & c & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}, \quad (3.19)$$

where  $abc > 0$  and, obviously,  $a + b + c = 1$ .

Complete the following tasks:

1. draw the probability transition diagram and describe the nature of the Markov chain;
2. determine the stability condition (if any) and, for a stable Markov chain, calculate the stationary state probabilities  $\pi_i, i \geq 0$ ;
3. by exploiting the results achieved in point 2 above, calculate the z-transform of the stationary state probabilities  $\pi_i, i \geq 0$ ;
4. for the specific case in which  $a = b = c = 1/3$  show whether the Markov chain is stable or not and, and if not stable, determine the nature of the Markov chain;
5. using only your intuition show the nature of the Markov chain under the assumptions that  $a = 1/4, b = 1/4, c = 2/4$ .

