EXERCISE 1: In some cases the embedded Markov Chain of an M/G/1 queueing system is characterized by the following transition matrix:

$$P = \begin{bmatrix} b_0 & b_1 & b_2 & b_3 & b_4 & \dots \\ a_0 & a_1 & a_2 & a_3 & a_4 & \dots \\ 0 & a_0 & a_1 & a_2 & a_3 & \dots \\ 0 & 0 & a_0 & a_1 & a_2 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \tag{3.19}$$

Complete the following tasks:

1. interpret elements a_i and b_i, $i \geq 0$, and describe (at least) one real system which can be modelled by an M/G/1 queuing system with the above transition matrix;
2. by means of stochastic arguments, prove that the steady probabilities π_i, $i \geq 0$, of the M/G/1 queuing system under examination satisfies the following (infinite) system of linear equations

$$\pi_i = \pi_0 b_i + \sum_{h=1}^{i+1} \pi_h a_{i-h+1}, \; i \in \mathrm{N} \tag{3.20}$$

3. prove that the z-transform $(\Pi(z))$ of the number of packets left in the system by a departing customer (i.e. at the embedding points) is given by

$$\Pi(z) = \pi_0 \frac{zB(z) - A(z)}{z - A(z)} \tag{3.21}$$

   where

$$B(z) = \sum_{i=0}^{\infty} b_i z^i, \; A(z) = \sum_{i=0}^{\infty} a_i z^i \tag{3.22}$$

4. derive the stability condition of the M/G/1 queuing system under examination and calculate, for a stable system, the steady state probability π_0.