Esercizio 2

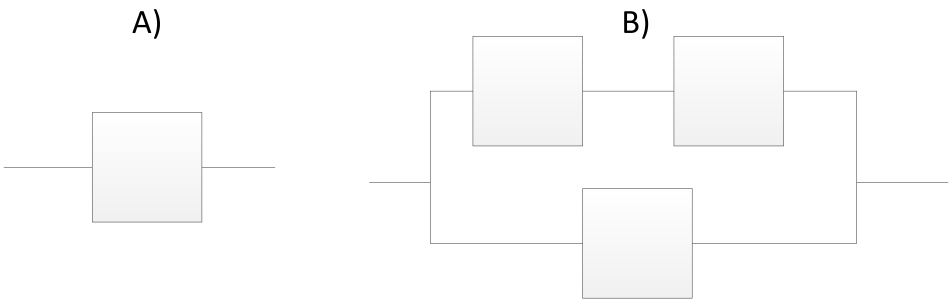
La ACME ha due diversi impianti di produzione di interruttori. Nell'impianto 1, ciascun pezzo presenta un difetto con probabilità $p_1 = 10^{-5}$, in modo indipendente dagli altri pezzi. Nell'impianto 2, si sa che il numero medio di pezzi difettosi in una settimana è pari a 5. La produzione è di $n = 4 \cdot 10^5$ pezzi alla settimana per impianto.

Il candidato:

1) Calcoli media e varianza del numero di pezzi difettosi prodotti dalla ACME in una settimana
2) Disegni, in maniera *qualitativa*, ma aggiungendo quanti più dettagli possibile, l'andamento della PMF del numero di pezzi difettosi prodotti dalla ACME in una settimana.
3) Calcoli la probabilità che il numero di pezzi difettosi prodotti dalla ACME in una settimana sia uguale a 5
4) Calcoli la probabilità che il numero di pezzi difettosi prodotti dalla ACME in una settimana sia minore di 3
5) Calcoli la probabilità che un pezzo preso a caso sia difettoso.

Si supponga adesso che i pezzi della ACME possano essere connessi in serie e/o in parallelo, e che il sistema che ne risulta sia funzionante se esiste una via che porta da un'estremità all'altra attraversando soltanto sistemi non difettosi. Il candidato:

6) Spieghi, giustificando la sua risposta, quale dei due sistemi a) e b) sotto riportati ha maggior probabilità di funzionare.



Esercizio 2

ACME components owns two switch production plants. In plant 1, each unit is defective with probability $p_1 = 10^{-5}$, independently from the others. In plant 2, the mean weekly number of defective units is equal to 5. The production of each plant is $n = 4 \cdot 10^5$ units per week.

1) Compute mean and variance of the number of defective units produced by ACME in a week.
2) Draw a *qualitative* plot (with as many details are possible) of the PMF of the number of defective units in a week.
3) Compute the probability that the weekly number of defective units produced by ACME is equal to 5.
4) Compute the probability that the weekly number of defective units produced by ACME is less than 3.
5) Compute the probability that a randomly chosen unit is defective.

Suppose now that ACME units can be connected in series or in parallel as above, and that the resulting system works if there exists a way that connects both extremities traversing only non-defective systems.

6) Explain which of the two systems has a higher chance to be functioning. Jusitfy your findings.

Solution

1) Given that *n* is large and *p* is small, we can approximate the failure probability of each plant using a Poisson variable, whose average is $\lambda_i = n_i \cdot p_i$. Hence, it is $\lambda_1 = 4, \quad \lambda_2 = 5$. Thus, there are on average 9 defective units in a weekly production of $2n = 8 \cdot 10^5$ pieces. As for the variance, it is all the more reasonable to approximate the whole production using a Poisson variable, whose average and variance is equal to 9.
2) The Poisson variable has a bell shape, with an infinite right tail. It peaks around its mean value, which is equal to 9. Hence, the shape is the following
3) The probability that 5 pieces are defective is equal to $p_5 = e^{-9} \cdot 9^5 / 5! = 0.060727$
4) The probability that less than 3 pieces are defective is equal to $p_0 + p_1 + p_2 = 1.23 \cdot 10^{-3} + 11.1 \cdot 10^{-3} + 49.98 \cdot 10^{-3} = 62.32 \cdot 10^{-3}$
5) The probability is the following:

$$\begin{aligned} p_d &= P\{\text{defective}\} \\ &= P\{\text{defective}|\text{plant 1}\} \cdot P\{\text{plant 1}\} + P\{\text{defective}|\text{plant 2}\} \cdot P\{\text{plant 2}\} \\ &= 10^{-5} \cdot 0.5 + \frac{5}{4 \cdot 10^5} \cdot 0.5 \\ &= 1.125 \cdot 10^{-5} \end{aligned}$$

6) System a) works with probability $p_a = 1 - p_d$. System b) works with probability

$$\begin{aligned} p_b &= 1 - P\{\text{upper branch fails}\} \cdot P\{\text{lower branch fails}\} \\ &= 1 - \left(1 - (1 - p_d)^2\right) \cdot p_d \end{aligned}$$

Thus, $p_b > p_a$ if and only if

$$1 - \left(1 - (1 - p_d)^2\right) \cdot p_d > 1 - p_d$$

$$p_d > \left(1 - (1 - p_d)^2\right) \cdot p_d$$

$$1 > 1 - (1 - p_d)^2$$

$$p_d < 1$$

which is always true. System b) is always more reliable than system a), no matter what the failure probability of a single component is.