ESERCIZIO 1: In some cases the embedded Markov Chain of an M/G/1 quaeing system is characterized by the following probability transition matrix:

$$P = \begin{bmatrix} b_0 & b_1 & b_2 & b_3 & b_4 & \dots \\ a_0 & a_1 & a_2 & a_3 & a_4 & \dots \\ 0 & a_0 & a_1 & a_2 & a_3 & \dots \\ 0 & 0 & a_0 & a_1 & a_2 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \quad (1.1)$$

Complete the following tasks:

1. interpret elements a_i and b_i, $i \geq 0$, and show (at least) one concrete example which gives rise to he above transition matrix;
2. by using stochastic motivations show that for the stationary state probabilitiess π_i, $i \geq 0$, the following system of linear equations holds:

$$\pi_i = \pi_0 b_i + \sum_{h=1}^{i+1} \pi_h a_{i-h+1}, \; i \in \mathrm{N} \quad (1.2)$$

3. prove that the z-transform $(\Pi(z))$ of the number of users (e.g. packets) in the system at the embedding points is given by:

$$\Pi(z) = \pi_0 \frac{zB(z) - A(z)}{z - A(z)} \quad (1.3)$$

   where

$$B(z) = \sum_{i=0}^{\infty} b_i z^i, \; A(z) = \sum_{i=0}^{\infty} a_i z^i \quad (1.4)$$

4. determine the system stability conditions and calculate, for a stable system, the stationary state probability π_0.