Exercise 2

Two bank clerks are assigned *standard* and *urgent* customers respectively. Let X denote the number of customers being attended to by the first clerk, and Y denote the number of customers of the second one at the same time. Let the JPMF of X and Y be the following:

X/Y	0	1	2	3
0	0.08	0.07	0.04	0.00
1	0.06	0.15	0.05	0.04
2	0.05	0.04	0.10	0.06
3	0.00	0.03	0.04	0.07
4	0.00	0.01	0.05	0.06

1) What is the probability that there is exactly one customer in each line?
2) What is the probability that the number of customers in the two lines are identical?
3) Let A denote the event that there are at least two more customers in one line than in the other line. Calculate the probability of A.
4) Determine the marginal PMF of X and then calculate the expected number of standard customers in line.
5) Determine the marginal PMF of Y.
6) Are X and Y independent random variables? Explain your answer.
7) Determine the PMF of the *overall* number of customers in line at the bank.

Esercizio 2

Due impiegati di banca hanno in consegna rispettivamente i clienti *standard* e *urgenti*. Sia X il numero di clienti in fila allo sportello standard, ed Y il numero di clienti in fila allo sportello "urgente" ad un dato momento del giorno. La JPMF delle due variabili aleatorie X ed Y è mostrata in tabella.
1) Qual è la probabilità che ci sia un cliente in entrambe le file?
2) Qual è la probabilità che il numero di clienti nelle due file sia identico?
3) Sia A l'evento "ci sono almeno due clienti in più in una delle due file rispetto all'altra". Si calcoli la probabilità di A.
4) Determinare la PMF marginale di X e calcolare il valore atteso di clienti in fila allo sportello standard.
5) Determinare la PMF marginale di Y
6) Spiegare se X ed Y sono indipendenti
7) Calcolare la PMF del numero *totale* di clienti in fila.

Solution

1) What is the probability that there is exactly one customer in each line?

$P(X=1;Y=1)=p(1,1)=0.15$

2) What is P(X = Y), that is, the probability that the number of customers in the two lines are identical?

$P(X=Y)=p(0,0)+p(1,1)+p(2,2)+p(3,3)=0.08+0.15+0.1+0.07=0.4$

3) Let A denote the event that there are at least two more customers in one line than in the other line. Calculate the probability of A.

$A=\{(x,y):x\geq y+2\}\cup\{(x,y):y\geq x+2\}$

$=\{(2,0),(3,0),(4,0),(3,1),(4,1),(4,2),(0,2)(0,3)(1,3)\}$

$P(A)=p(2,0)+p(3,0)+p(4,0)+p(3,1)+p(4,1)+p(4,2)+p(0,2)+p(0,3)+p(1,3)=0.22$

4) Determine the marginal PMF of X and then calculate the expected number of customers in the standard queue.

$p_X(n)=\sum_{i=-\infty}^{+\infty}p(n,i)=p(n,0)+p(n,1)+p(n,2)+p(n,3)$

x	0	1	2	3	4
px(x)	0.19	0.30	0.25	0.14	0.12

Hence, $E(X)=\sum_{x=1}^{4}x\cdot p_X(x)=1\cdot 0.19+2\cdot 0.25+3\cdot 0.14+4\cdot 0.12=1.7$

5) Determine the marginal PMF of Y .

$p_Y(n)=\sum_{i=-\infty}^{+\infty}p(i,n)=p(0,n)+p(1,n)+p(2,n)+p(3,n)+p(4,n)$

Y	0	1	2	3
$p_Y(y)$	0.19	0.30	0.28	0.23

6) Are X and Y independent random variables? Explain.

They are not. By counterexample: $P(X=4)=0.12$, $P(Y=0)=0.19$, $P(X=4;Y=0)=0.$

7) Determine the PMF of the *overall* number of customers in line at the bank.

You only need to sum up the cells in the above table whose overall number of customers is the same. The following PMF is obtained.

s	p(s)
0	0.08
1	0.13
2	0.24
3	0.09
4	0.17
5	0.11
6	0.12
7	0.06